Practical Tests for Sub-Rayleigh Source Discriminations with Imperfect Demultiplexers

Konrad Schlichtholz, Tomasz Linowski, Łukasz Rudnicki
International Centre for Theory of Quantum Technologies, University of Gdansk, 80-308 Gdańsk, Poland
POLAND

Mattia Walschaers, Nicolas Treps
[2]Laboratoire Kastler Brossel, Sorbonne Université, ENS-Université PSL, CNRS, Collège de France, 4 Place Jussieu, 75252 Paris
FRANCE

Giacomo Sorelli
Fraunhofer IOSB, Ettlingen, Fraunhofer Institute of Optronics,
System Technologies and Image Exploitation, Gutleuthausstr. 1, 76275 Ettlingen, Germany
GERMANY
giacomo.sorelli@iosb-extern.fraunhofer.de

ABSTRACT

Quantum-optimal discrimination between one and two closely separated light sources can be theoretically achieved by ideal spatial-mode demultiplexing, simply monitoring whether a photon is detected in a single antisymmetric mode. However, we show that for any imperfections of the demultiplexer, no matter how small, this simple statistical test becomes practically useless. While we identify a class of separation-independent tests with vanishing error probabilities in the limit of large numbers of detected photons, they are generally unreliable beyond that very limit. As a practical alternative, we propose a simple semi-separation-independent test, which provides a method for designing reliable experiments, through arbitrary control over the maximal probability of error.

1.0 INTRODUCTION

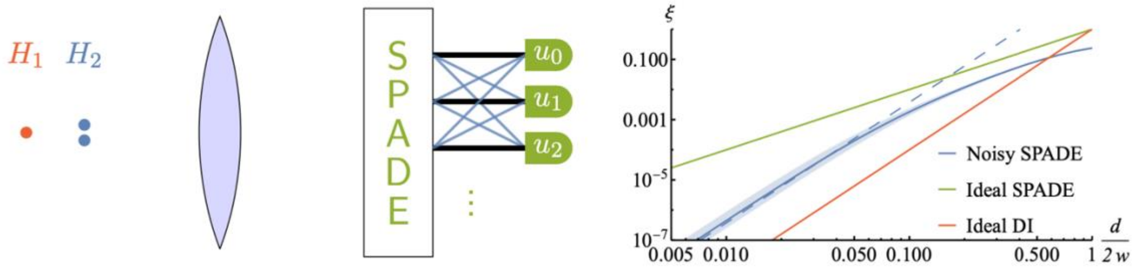
The analysis of images acquired through optical systems is a fundamental tool for our understanding of the world both at microscopic and macroscopic length scales. On the one hand, improving the resolution of optical microscopes allows to enhance our diagnostical capability. On the other hand, large diffraction-limited telescopes play a crucial in remote sensing and surveillance, e.g., in obtaining high-resolution images of artificial satellites or in imaging the surface of Earth from space. Therefore, quantum inspired techniques to improve optical resolution are relevant to defense and security and should be discussed during this symposium.

The resolution of an imaging system can be defined by its ability of discriminating whether an image originates from one or two light sources. Historical resolution criteria, e.g. that of Rayleigh, tell us that, for source separations smaller than the width of the point spread function of the imaging system, the efficiency of source discrimination protocols based on spatially resolved intensity measurements, i.e. direct detection, drops significantly. Recent works, using tools of quantum detection and estimation theory, proved that sub-Rayleigh resolution can be achieved by replacing direct detection, e.g. with a camera, with spatial-mode demultiplexing (SPADE) [1,2]. In particular, it was demonstrated that, due to the symmetry of the problem, detecting even just one photon in a fixed antisymmetric mode allows to accept one of the hypotheses with zero probability of error, leading to a near-optimal decision strategy [2]. However, these findings were obtained assuming ideal measurements, while real devices are always affected by imperfections. In particular, in the case of SPADE, misalignment, defects in the fabrication of the demultiplexer and other imperfections induce a finite probability of detecting photons in the incorrect output, i.e. crosstalk.

In this contribution, we discuss how crosstalk has a huge impact on SPADE in discriminating between one and two sources. In particular, we show that the simple test discussed above goes from being close to optimal in the ideal case, to become as good as flipping a coin in presence of arbitrarily small imperfection. To obviate this problem, using quantum-inspired techniques, we propose new statistical tests that allow to achieve sub-Rayleigh resolution even in presence of imperfections [3].

2.0 METHODS AND RESULTS

Figure 1: Problem illustration and Chernoff exponent.



(Right) Schematic representation of the measurement scenario. Depending on the hypothesis, there is one (H1) or two (H2) weak light sources in the object plane, resulting in diffraction-broadened spatial field distributions in the image plane. To decide whether H1 or H2 is true the image-plane field distribution is analysed via photon counting after spatial-mode demultiplexing affected by crosstalk. (Left) Comparison between the Chernoff exponents for SPADE ξ in presence of crosstalk (blue), asymptotic Chernoff exponent for ideal direct imaging ξDI (red) and the quantum bound ξQ obtained by ideal SPADE (green) versus d/ 2w.

We want to discriminate between two hypotheses, H1 and H2, as illustrated in Figure 1. Under hypothesis H2, two incoherent light sources of equal intensity are separated by a distance d in the object plane. According to hypothesis H1, only one source is present in the object plane and has the same total intensity of the two sources from hypothesis H2. How good a given strategy for deciding between the two hypotheses is quantified by the average probability of error after detecting N photons: $P_e(N) = (\alpha(N) + \beta(N))/2$, where $\alpha(N), \beta(N)$ are the probabilities of error of the first and second kind, i.e. assuming H2 when H1 is correct and vice versa.

Effective source discrimination for sub-Rayleigh separations, i.e. for $x := d/2w < 1$, with w the width of the point spread function of the imaging system, requires to detect a large number N of photons. When $N >> 1$, the probability of error, minimized over all possible decision strategies for a specific measurement, decays exponentially as $P_e(N) = e^{-\xi N}$, where ξ is the Chernoff exponent. By maximising the Chernoff exponent over all possible measurements allowed by quantum mechanics, we obtain the *quantum Chernoff exponent* ξ_Q, which provides the smallest achievable error probability. For our discrimination task, the quantum Chernoff exponent scales like $\xi_Q \sim x^2$, and is achieved by intensity measurements after spatial mode demultiplexing (SPADE). On the contrary, by performing spatially distributed intensity measurements, i.e. direct imaging (DI), as one traditionally does with a camera, one only achieve the less favourable scaling $\xi \sim x^4$. Furthermore, SPADE allows optimal discrimination with a very simple statistical test: we monitor a single asymmetric spatial mode and record the total number N_1 of photons in that mode, if $N_1 > 0$, we conclude that there are two sources [2].

Unfortunately, in practical scenarios, demultiplexer are imperfect, and there is always a finite possibility of detecting photons in the wrong mode, i.e. crosstalk. This crosstalk affects the Chernoff exponent (see Figure 1-1, left), but for values reported in typical experiments [4, 5] crosstalk-affected SPADE can still outperform DI. However, the simple statistical test discussed above cannot be used anymore. In fact, this statistical test has a probability of error $P_e(N)=1/2$ (it become as good as flipping a coin) for arbitrarily small amounts of crosstalk [3].

To obviate this problem, we constructed a new statistical test based on the number of photons N_1 in the same antisymmetric mode, which asymptotically approaches the optimal error probability as given by the Chernoff exponent (blue line in Figure 1-1). Such tests consist in comparing N_1 with a threshold Λ. If $N_1>\Lambda$, we decide that there are two sources, otherwise we decide that there is only one source. The threshold reads $\Lambda = N[p_0+\gamma(d_0/w)^2]$, with γ a constant of the order of unity, and p_0 the probability of crosstalk into the mode we are monitoring [3].

3.0 CONCLUSIONS

We analyzed the efficiency of realistic SPADE-based discrimination between one and two closely-separated light sources. We demonstrated that the presence of crosstalk impacts the probability of successful discrimination, and more importantly that any crosstalk requires the design of new statistical tests. We therefore proposed a new discrimination algorithm, which, even for imperfect demultiplexers, gives access to the minimal error probability.

This is a first step toward the use of SPADE in real discrimination tasks and pave the way for the study of more complex classification tasks. Bringing the performances of the latter to the minimal error probabilities enabled by quantum mechanics could have a significant impact defense and surveillance.

4.0 REFERENCES

[1] M. Tsang, R. Nair, and X.-M. Lu, Phys. Rev. X 6, 031033 (2016).

[2] X.-M. Lu, H. Krovi, R. Nair, S. Guha, and J. H. Shapiro, npj Quantum Information 4, 64 (2018).

[3] Konrad Schlichtholz, Tomasz Linowski, Mattia Walschaers, Nicolas Treps, Łukasz Rudnicki, Giacomo Sorelli, arXiv: 2303.02654 (2023).

[4] Clémentine Rouvière, David Barral, Antonin Grateau, Ilya Karuseichyk, Giacomo Sorelli, Mattia Walschaers, Nicolas Treps, arXiv:2306.11916 (2023)

[5] Pauline Boucher, Claude Fabre, Guillaume Labroille, and Nicolas Treps, Optica 7, 1621-1626 (2020)